

Using Wireless Network Coding to Replace a Wired with Wireless Backhaul

Henning Thomsen, Elisabeth de Carvalho, Petar Popovski
 Department of Electronic Systems, Aalborg University, Denmark
 Email: {ht,edc,petarp}@es.aau.dk

Abstract—Cellular networks are evolving towards dense deployment of small cells, which requires flexible and efficient backhauling solutions. A viable solution that reuses the same spectrum is wireless backhaul where the Small Base Station (SBS) acts as a relay. In this paper we consider a reference system that uses wired backhaul and each Mobile Station (MS) in the small cell has its uplink and downlink rates defined. The central question is: *if we remove the wired backhaul, how much extra power should the wireless backhaul use in order to support the same uplink/downlink rates?* We introduce the idea of wireless-emulated wire (WEW), based on two-way relaying and network coding. This setup leads to a new type of broadcast problem, with decoding conditions that are specific to the requirement for equivalence to the wired backhaul. We formulate and solve the associated optimization problems. The proposed approach is a convincing argument that wireless backhauling solutions should be designed and optimized for two-way communication.

Index Terms—Wireless Network Coding, Heterogeneous Networks, Small-cell networks, Beamforming

I. INTRODUCTION

The next leap in increasing the wireless data rates for multiple users is bringing the access point closer to the users and enable spatial reuse over smaller distances. Such a *network densification* [1] is a key feature of the upcoming 5G systems. The trend of densification has already started with the deployment of femtocells (small cells). The proliferation of small cells is conditioned on the existence of a flexible, cost-effective backhaul solution [2] between a Base Station (BS) and a Small-cell Base Station (SBS), which can permit rapid deployment of SBSs. Wired backhaul does not use spectrum, but is not always feasible due to cost or even not possible at all, as it is the case with nomadic cells with SBS on a car [3].

We consider a wireless backhaul link BS-SBS that uses the same spectrum as the link Mobile Station (MS)-SBS, such that SBS acts as an in-band relay. During the last decade, a relay has been seen as an enabler of improved coverage [4], but Wireless Network Coding (WNC) has introduced a fresh potential by exhibiting gains in spectral efficiency for scenarios with two-way relaying [5]–[7]. Perhaps the most important message of WNC is that the design of two-way communication schemes, rather than decoupling the uplink and downlink traffic, significantly expands the space of communication strategies and the potential gains. This has been shown in [8]. In [9], the authors study cooperative communication strategies which use network coding and beamforming, in a relay-aided two-source two-sink network, and a backhaul between the sources. Rate gains are shown in the schemes using backhaul.

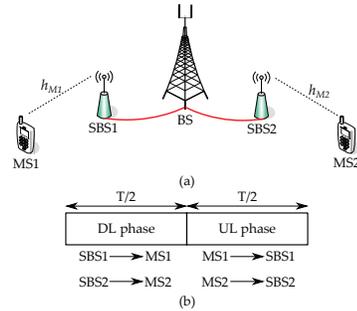


Figure 1. (a) A reference system with two small cells. (b) The transmissions in the uplink and downlink; only the wireless transmissions are depicted, the transmissions over the wired backhaul are taking place in parallel.

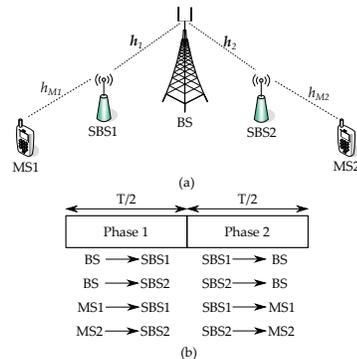


Figure 2. System Model used in this paper, where the two SBSs are antipodal, and each of them serve two-way traffic flow of the MS.

The objective of this letter is to show how wired backhaul can be replaced by a wireless one without using additional spectrum and by reusing the existing modulation/coding over the wireless air interface. The simplest version of the problem can be explained using Figs. 1 and 2. We first ignore SBS2/MS2 and consider only two-way communication between BS and MS1 through SBS1. The system is based on Time Division Duplex (TDD) with a time frame of duration T , such that $\frac{T}{2}$ is allocated to the Downlink (DL) and $\frac{T}{2}$ to the uplink (UL) transmission, respectively. The transmission rates of MS1 on Fig. 1 are R_{D1}/R_{U1} in DL/UL. These rates are supported by the backhaul. The DL and UL rates averaged over the whole interval T , are $(\frac{R_{D1}}{2}, \frac{R_{U1}}{2})$. The central question in this letter is: *Can we remove the wired backhaul and still support the same rate pair $(\frac{R_{D1}}{2}, \frac{R_{U1}}{2})$ within the interval of length T , without requiring any changes in the baseband of MS1?* To show that this is indeed the case, we leverage

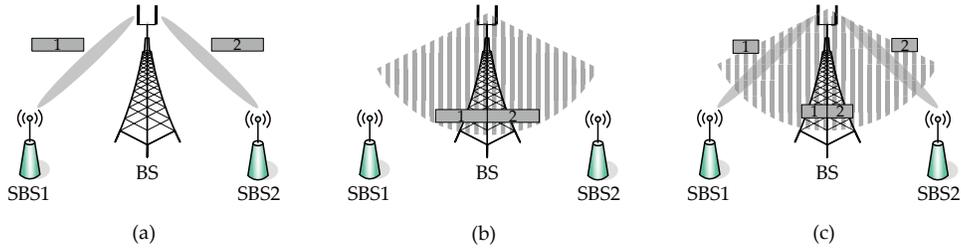


Figure 3. The three transmission methods considered in this paper. In (a), the BS transmits to the SBSs using ZF. In (b), the BS broadcasts the concatenated data to both SBSs. The WEW is shown in (c), wherein the BS transmits data using both ZF, and the common beamforming.

the idea of two-way relaying with XOR-WNC and devise a two-phase transmission. In Phase 1, BS and MS1 transmit simultaneously. MS1 transmits with the same power and rate R_{U1} , as in Fig. 1. BS now needs to use power P_B for the wireless transmission at rate R_{D1} . Assuming that R_{U1} is equal to the capacity of the link MS1-SBS1, then we should ensure that SBS1 decodes the “clean” signal of MS1, without any residual interference from the BS. Therefore, the minimal P_B should allow SBS1 to decode the signal of rate R_{D1} from the BS by treating the signal from MS1 as noise, then cancel the signal from the BS and proceed to decode the signal of rate R_{U1} from MS1. In Phase 2, SBS1 XORs the decoded messages and broadcasts them to MS1 and BS, such that at the end of T , the performance is equivalent to the wired backhaul i.e. we have obtained a *wireless-emulated wire (WEW)*.

This letter treats a more advanced version of the problem, where a multi-antenna BS has two simultaneous two-way links to MS1 and MS2 through SBS1 and SBS2, respectively. The first phase gives rise to a broadcast problem with a new type of constraint, since each SBS i needs to remove all the interference from the BS before decoding the signal from MS i . This is different from the conditions in standard broadcast or interference channels and we propose a new transmission technique in order to address the problem. Although the letter treats the case of only $L = 2$ two-way links, the approach can be generalized to an arbitrary number of two-way links.

II. SYSTEM MODEL AND SCHEME DESCRIPTION

The wired scenario is shown in Fig. 1, and the wireless in Fig. 2. Here, MS i is connected to SBS i through the channel $h_{Mi} \in \mathbb{C}$, while SBS i is connected to BS by wire. For convenience, we denote the wireless channel from BS to SBS i by $h_i^* \in \mathbb{C}^{[2M \times 1]}$, where $(\cdot)^*$ is complex conjugation. All channels are reciprocal and constant through the entire transmission. MS i and SBS i are single-antenna nodes, while BS has $2M$ antennas. BS transmits at power P_B , SBS i at P_{Si} , and MS i at P_{Mi} . All nodes are half-duplex, and full Channel State Information is assumed. The small cells are spatially separated, causing negligible interference to each other.

The bandwidth-normalized capacity of the link is $C(\gamma) = \log_2(1 + \gamma)$, where γ is the Signal-to-Noise Ratio (SNR). Denoting the noise power by σ^2 , in the wired case we can define the UL and the DL SNR as $\gamma_{Ui} = \frac{P_{Mi}|h_{Mi}|^2}{\sigma^2}$ and $\gamma_{Di} = \frac{P_{Si}|h_i|^2}{\sigma^2}$, respectively. The uplink and the downlink

rates, chosen for the wired backhaul, are equal to the capacity $R_{Ui} = C(\gamma_{Ui})$ and $R_{Di} = C(\gamma_{Di})$, respectively. In order to preserve equivalent rates for the MS i , the parameters from the wired backhaul case that are kept to be identical for the wireless backhaul are h_{Mi} , P_{Mi} , as well as the rates R_{Ui} and R_{Di} .

Our WEW scheme is designed to benefit from two transmission options at the BS. A natural choice is to use Zero Forcing (ZF) beamforming at the BS and send data to the two SBSs through two orthogonal spatial channels. Each SBS i receives only its intended message from the BS. This is shown in Fig. 3(a), where message 1 is sent to SBS1, and message 2 to SBS2. Because the message sent by ZF is received only by the intended SBS, it is referred to as a *private* message. This terminology is inspired by the Han-Kobayashi scheme in interference channels [10], but the reader should note that our setup is completely different.

ZF beamforming is detrimental when the channel is ill-conditioned, resulting in noise enhancement at low SNR. A viable alternative could be MMSE beamforming; however it cannot be used as it leaves a residual interference at the SBS that cannot be decoded, hence violating our condition that MS should be able to send to SBS over a “clean” channel as with wired backhaul. In the extreme case where the channels h_1 and h_2 are collinear, a better solution is to send a *common* message that is broadcasted to both receivers, see Fig. 3(b): the data bits of the two messages are concatenated at the BS into a common message, which is then encoded. Both SBSs must decode this common message in its entirety, and therefore remove its contribution before decoding the UL signal.

In WEW, the BS splits the message for each MS into a private and common part, see Fig. 3(c). The private part is sent using ZF. The common parts of the messages to both MSs are concatenated and sent using a common beam. SBS1 must decode the private message 1 (gray narrow beams) and the common message (wide patterned beam). SBS2 is treated analogously. The transmission is carried out in two phases.

1) *Phase 1*: The BS transmits the downlink message for MS i , to SBS i at rate R_{Di} . The message consists of NR_{Di} bits, where N is the total number of channel uses in a slot. The message is split into a private data containing $N\alpha_i R_{Di}$ bits and a common data containing $N(1 - \alpha_i)R_{Di}$ bits, where $0 \leq \alpha_i \leq 1$, $i = 1, 2$ is the splitting factor that is subject to optimization. We define the private rate $R_{Pi} = \alpha_i R_{Di}$ and the

common rate $R_{Ci} = (1 - \alpha_i) R_{Di}$. The common data parts for both users are concatenated in order to obtain a single common message consisting of $N(R_{C1} + R_{C2}) = NR_C$ bits. Each of the three messages, two private and one common, are now encoded separately, using random Gaussian codebooks with appropriate rates, such that the signal sent by the BS is:

$$\mathbf{x}_B = \sqrt{P_1} \mathbf{w}_1 x_1 + \sqrt{P_2} \mathbf{w}_2 x_2 + \mathbf{w}_C x_C, \quad (1)$$

where x_i is private message for MS i with rate R_{Pi} , and x_C is the common message with rate R_C . The private message is sent using power P_i . The beamformers $\mathbf{w}_i \in \mathbb{C}^{[2M \times 1]}$ are defined using the ZF condition, i.e. they must satisfy $\mathbf{h}_i^H \mathbf{w}_j = 0, i, j = 1, 2, i \neq j$. For the two-stream case, $\mathbf{w}_i = (I_{2M} - (\mathbf{h}_j^* \mathbf{h}_j^H) / |\mathbf{h}_j|^2) \mathbf{h}_i^*$, for $i, j = 1, 2, i \neq j$ [11]. Here I_{2M} is the $2M \times 2M$ identity matrix and $(\cdot)^H$ is Hermitian transpose. For analytical convenience, \mathbf{w}_i is normalized. The beamformer $\mathbf{w}_C \in \mathbb{C}^{[2M \times 1]}$ is used for the common message, sent at power $P_C = \|\mathbf{w}_C\|^2$, and is found in Sec. III.

Simultaneously, MS i transmits x_{Mi} . SBS i then receives

$$\begin{aligned} y_{Si} &= \mathbf{h}_i^H \mathbf{x}_B + h_{Mi} x_{Mi} + z_{Si} \\ &= \mathbf{h}_i^H \sqrt{P_i} \mathbf{w}_i x_i + \mathbf{h}_i^H \mathbf{w}_C x_C + h_{Mi} x_{Mi} + z_{Si}, \end{aligned} \quad (2)$$

where we use the ZF condition and z_{Si} is the AWGN at SBS i .

2) *Phase 2:* After SBS i has decoded the private and the common message, it recreates the original message for MS i , computes the XOR of that message and the message decoded from the MS i and broadcasts the result. BS, MS1 and MS2 decode the signal sent by their respective SBS i and apply XOR to recover the desired message from the broadcasted message.

III. OPTIMIZATION PROBLEMS

We first look at the minimization of the the transmit power at the BS, subject to the given UL/DL rate constraints. The variables are the powers P_i for the ZF beamformers, the common beamformer \mathbf{w}_C and its power $\|\mathbf{w}_C\|^2$, as well as the splitting factors α_i , for $i = 1, 2$.

For the constraints, we have a Multiple Access Channel (MAC) region at each SBS i , involving the rates R_{Pi} , R_{Ci} and their sum. Formally, this is a 3-user MAC channel with a three-dimensional achievable region. However, x_i is sent at a rate equal to the single-user capacity, i.e. $R_{Ui} = C(\gamma_{Ui})$, which determines a specific two-dimensional sub-region of the three-dimensional MAC region. Then the constraints not involving R_{Ui} can be dropped, since they will be fulfilled if the corresponding constraints with R_{Ui} are fulfilled. The UL message is treated as noise when decoding x_i and x_C . Writing $\overline{\gamma}_{Pi} = \frac{P_i |\mathbf{h}_i^H \mathbf{w}_i|^2}{\sigma^2 (1 + \gamma_{Ui})}$, $\overline{\gamma}_{Ci} = \frac{|\mathbf{h}_i^H \mathbf{w}_C|^2}{\sigma^2 (1 + \gamma_{Ui})}$ and $\overline{\alpha}_i = 1 - \alpha_i$, and recalling $R_{Pi} = \alpha_i R_{Di}$ and $R_{Ci} = \overline{\alpha}_i R_{Di}$, the optimization problem is:

$$\begin{aligned} & \text{minimize}_{P_i, \mathbf{w}_C, \alpha_i} && P_1 + P_2 + \|\mathbf{w}_C\|^2 \\ & \text{subject to} && \alpha_i R_{Di} \leq C(\overline{\gamma}_{Pi}) \\ & && \overline{\alpha}_1 R_{D1} + \overline{\alpha}_2 R_{D2} \leq C(\overline{\gamma}_{Ci}) \\ & && \alpha_i R_{Di} + \overline{\alpha}_1 R_{D1} + \overline{\alpha}_2 R_{D2} \leq C(\overline{\gamma}_{Pi} + \overline{\gamma}_{Ci}) \\ & && P_i \geq 0, \quad 0 \leq \alpha_i \leq 1, i = 1, 2. \end{aligned}$$

Let us define $\beta_{1i} = \sigma^2 (2^{R_{Pi}} - 1) (1 + \gamma_{Mi})$, $\beta_{2i} = \sigma^2 (2^{R_C} - 1) (1 + \gamma_{Mi})$, and $\beta_{3i} = \sigma^2 (2^{R_{Pi} + R_C} - 1) (1 + \gamma_{Mi})$. The problem then becomes

$$\begin{aligned} & \text{minimize}_{P_i, \mathbf{w}_C, \alpha_i} && P_1 + P_2 + \|\mathbf{w}_C\|^2 \\ & \text{subject to} && \beta_{1i} \leq P_i |\mathbf{h}_i^H \mathbf{w}_i|^2 \\ & && \beta_{2i} \leq |\mathbf{h}_i^H \mathbf{w}_C|^2 \\ & && \beta_{3i} \leq P_i |\mathbf{h}_i^H \mathbf{w}_i|^2 + |\mathbf{h}_i^H \mathbf{w}_C|^2 \\ & && P_i \geq 0, \quad 0 \leq \alpha_i \leq 1, i = 1, 2 \end{aligned}$$

The second and third constraints are not convex and we rewrite the problem using Semidefinite Programming (SDP) [12]. Let $\mathbf{H}_i = \mathbf{h}_i \mathbf{h}_i^H$ and $\mathbf{W}_C = \mathbf{w}_C \mathbf{w}_C^H$. We can write $\|\mathbf{w}_C\|^2 = \text{Tr}(\mathbf{w}_C \mathbf{w}_C^H) = \text{Tr}(\mathbf{W}_C)$, where Tr is the trace of a matrix. Also, $|\mathbf{h}_i^H \mathbf{w}_C|^2 = \text{Tr}(\mathbf{h}_i \mathbf{h}_i^H \mathbf{w}_C \mathbf{w}_C^H) = \text{Tr}(\mathbf{H}_i \mathbf{W}_C)$. The problem is then rewritten to

$$\begin{aligned} & \text{minimize}_{P_i, \mathbf{W}_C, \alpha_i} && P_1 + P_2 + \text{Tr}(\mathbf{W}_C) \\ & \text{subject to} && \beta_{1i} \leq P_i |\mathbf{h}_i^H \mathbf{w}_i|^2 \\ & && \beta_{2i} \leq \text{Tr}(\mathbf{H}_i \mathbf{W}_C) \\ & && \beta_{3i} \leq P_i |\mathbf{h}_i^H \mathbf{w}_i|^2 + \text{Tr}(\mathbf{H}_i \mathbf{W}_C) \\ & && \mathbf{W}_C \succeq 0, \quad \text{Rank}(\mathbf{W}_C) = 1 \\ & && P_i \geq 0, \quad 0 \leq \alpha_i \leq 1, i = 1, 2. \end{aligned}$$

The constraint $\mathbf{W}_C \succeq 0$ means that \mathbf{W}_C is positive semidefinite. This problem is not convex because of the rank one constraint [12]. By dropping this constraint, we obtain a lower bound on the objective, since the feasible set is enlarged. This problem can then be solved using SDP. Given the solution \mathbf{W}_C , we write it as $\mathbf{W}_C = \lambda_1 \mathbf{v}_1 \mathbf{v}_1^H + \lambda_2 \mathbf{v}_2 \mathbf{v}_2^H$, where $\lambda_1 \geq \lambda_2 \geq 0$ are the eigenvalues of \mathbf{W}_C , with corresponding eigenvectors $\mathbf{v}_1, \mathbf{v}_2$. When the solution \mathbf{W}_C is rank 1 ($\lambda_2 = 0$), the solution is then $\mathbf{w}_C = \sqrt{\lambda_1} \mathbf{v}_1$. Otherwise ($\lambda_2 \neq 0$), we approximate the solution by $\sqrt{\lambda_1} \mathbf{v}_1$, provided it is feasible. If it is not, $\sqrt{\lambda_1} \mathbf{v}_1$ is scaled to make it feasible.

We now look at the minimal transmission power of SBS. The total transmission power of the SBSs is $P_{S1} + P_{S2}$ in the wired case. For simplicity, assume that for the wired backhaul, $P_{S1} = P_{S2} = P_S$, while for the wireless backhaul the power of SBS i is $\eta_i P_S$, where $\eta_i \geq 1$, such that the extra power compared to the wired case is $(\eta_1 - 1)P_S + (\eta_2 - 1)P_S$. When both SBSs transmit to the BS, the rate region at the BS is a two-sender Single-Input Multiple-Output MAC. The problem is then

$$\begin{aligned} & \text{minimize}_{\eta_1, \eta_2} && \eta_1 + \eta_2 \\ & \text{subject to} && R_{D1} \leq \log_2 |I_{2M} + \eta_1 P_{S1} \mathbf{H}_1 / \sigma^2| \\ & && R_{D2} \leq \log_2 |I_{2M} + \eta_2 P_{S2} \mathbf{H}_2 / \sigma^2| \\ & && R_{D1} + R_{D2} \\ & && \leq \log_2 |I_{2M} + (\eta_1 P_{S1} \mathbf{H}_1 + \eta_2 P_{S2} \mathbf{H}_2) / \sigma^2| \\ & && \eta_1 \geq 1, \quad \eta_2 \geq 1. \end{aligned}$$

where $|\mathbf{X}| = \det(\mathbf{X})$ and with extra power $P_S(\eta_1 + \eta_2 - 2)$.

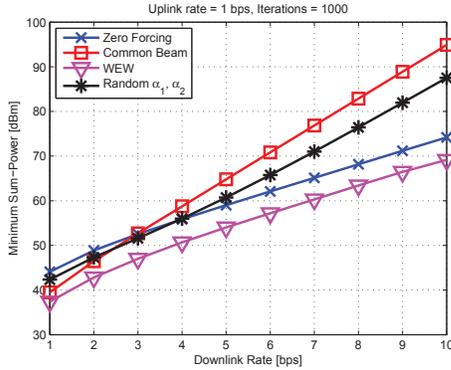


Figure 4. Comparison of minimum transmission power at BS, for the methods considered in this work.

IV. NUMERICAL RESULTS

The performance of WEW is demonstrated in this section. We assume that h_1, h_2 are Rayleigh faded. We set $M = 1$, so the BS has two antennas. The optimization problems in Sec. III are solved using the optimization software CVX. R_{U_i} and R_{D_i} are set as simulation parameters and we assume $R_{U_1} = R_{U_2}$ and $R_{D_1} = R_{D_2}$. The results are averaged over 1000 channel realisations. The bandwidth is normalized to 1 Hz. In the simulations, the common beamformer is obtained using the approximation described in Sec. III. In practically all cases, W_C is rank 1, so we can extract the solution $\sqrt{\lambda_1}v_1$ directly.

In Fig. 4, we compare WEW to using only ZF, only common beam, and random selection of the splitting factors. We set $R_{U_i} = 1$ bps, and vary R_{D_i} between 1 and 10 bps. It is observed that WEW has an advantage of about 6 dBm over the other methods, in a large part of the range. Also, the optimization of α_1 and α_2 results in a gain over random selection. It can be noted that there is a crossing point when $R_{D_i} = 4$ bps. For lower rates, the common beamformer has better performance, while ZF is better for higher rates. This is because ZF beamforming has an advantage at high SNRs, since low rate requirements translate into low SNR requirements. In Fig. 5, the results of the power optimization of SBS is shown. We assume that the SBS-BS channels are statistically equal to the MS-SBS channels. As expected, the required extra power increases with the rate of the XORed packet, as it now needs also to be sent to the BS.

V. CONCLUSION

In this paper, we have proposed Wireless-Emulated Wire (WEW), a concept that enables efficient wireless backhaul for two-way traffic. At the Base Station (BS), our scheme leverages on transmitting data to the Mobile Stations (MSs) by partitioning it into a private and common part, that should be decoded by the respective Small BS (SBS). We formulated an optimization problem to find the minimal transmission power at the BS, given the rates of the wired backhaul. Due to nonconvexity, the problem was relaxed and a lower bound on the required power was found. We have also investigated the additional power required at each SBS to support wireless

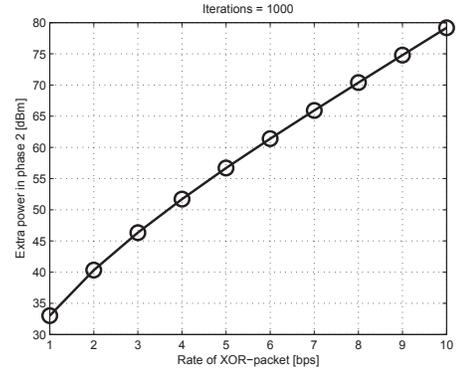


Figure 5. Total additional power required at the SBSs in phase 2.

backhaul. The WEW concept opens up further challenges in resource allocation when more than two MSs are considered.

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